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Visualization of Wave Propagation in Muffler

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***Abstract*:** This paper presents a prediction technique for wave propagation in muffler using Boundary Element Method (BEM). The results of the numerical calculation are compared with the plane-wave theory and also the experimental results. It is shown that at high frequencies the plane-wave theory does not yield correct results due to the occurrence of two- and three-dimensional wave motion. On the other hand, the BEM is able to predict the acoustic wave behavior in the high frequencies where the plane-wave theory fails.

The BEM uses a multi-domain technique to model complex mufflers, which may consist of cavities, tubes, perforates, and porous materials. Contour plots of sound pressure level inside the mufflers obtained from the BEM results are used to illustrate the principle of sound cancellation and to visualize the wave propagation behavior.

***Keywords*:** muffler, BEM, wave, visualization, noise.

1. Introduction

In many applications in industry, mufflers are used for noise reduction by attenuating the radiated sound through reactive effects (i.e. cancellation). The effect of a change in the cross section of a circular duct on the reflection of sound waves was first described by Miles (1944); he determined the reflection coefficient due to a discontinuity by calculating the pressure distribution in the vicinity of the discontinuity. Davies et al. (1954) derived an equation for the transmission loss of an "expansion-chamber," but their analysis considered only plane waves. Therefore, its validity is limited to low frequencies where all modes except the lowest mode are cut off. Alfredson (1972) investigated the effect of a continuous variation of the duct cross section by dividing the duct into a series of uniform subsections. His analysis was limited to nonspinning modes. A calculation of the transmission loss of expansion-chamber mufflers was presented by Young and Crocker (1975) using finite element method. They showed that the stability of the solution depends on the number of elements considered. Application of the finite element method to study the performance of some simple muffler elements was conducted by Craggs (1976), in which a general hexahedral element was used. Sharkawy and Nayfeh (1978) investigated the effect of an expansion-chamber silencer on the sound propagation and reflection in circular ducts, in which an analytical and experimental study were presented and the effects of expansion ratio, chamber length, and sound frequency were illustrated.

Although theories have been presented for the reactive effects, virtually all practical "reactive" mufflers also contain some elements that attenuate sound through dissipation, that is, by the conversion of sound energy to heat through the principle of viscosity. Porous materials such as glasswool or open-cell foams are commonly used in acoustics to absorb sound through mainly viscous effects. However, such materials cannot be used in many mufflers where the flow is of high temperature, wet, or contains material that may be imbedded in the porous material. Instead, these mufflers use perforated tubes to create dissipation through interaction with flow.

A variety of techniques of muffler modeling and experimental methods were presented by Munjal (1987). Most muffler designs are based on either lumped-parameter or one-dimensional (plane-wave) models. These models work reasonably well for low frequencies, i.e. when the wavelength of interest is larger than the cross-sectional dimension of the muffler. However, at higher frequencies, two- and three-dimensional wave motion inside the chamber "short circuits" the muffler by allowing sound intensity through to the exit of the muffler. This effect cannot be predicted by plane-wave theory. The onset of higher-order acoustic modes in the chamber causes precipitous drop in the measured transmission loss at high frequencies.

In this paper, the Boundary Element Method (BEM) is used to model complex mufflers that may contain cavities, tubes, perforates, and porous materials. The predicted transmission loss by the BEM is compared with plane-wave theory calculation and available experimental results. Contour plots of sound pressure level inside the mufflers obtained from the BEM results are used to illustrate the principles of sound cancellation and to understand why a specific muffler works (or does not work).

2. The Boundary Element Method (BEM)

The major advantage of the Boundary Element Method (BEM) is the reduction of the dimension of the problem being dealt with (Soenarko, 1983; Seybert et al., 1985). Using the BEM the three-dimensional problem is solved by applying two-dimensional treatment on the surface of the body in question. More advantage can be obtained when the body is of axisymmetric type (Seybert et al., 1986; Soenarko, 1993), in which the three-dimensional problem is further reduced to one-dimensional problem by the reduction of surface integral to line integral along the generator of the body.

The Boundary Element Method (BEM) is based on the Helmholtz Integral equation (Soenarko, 1983; Seybert et al., 1985):

$$C(P)p(P) = \int_S [\, p(Q)G'(P,Q) + ikz_0 v(Q)G(P,Q)]dS(Q) \qquad (1)$$

where S is the surface (boundary) of any arbitrary body or cavity of interest, $p(P)$ is the unknown sound pressure at any point P; $v(Q)$ is the acoustic particle velocity (equals zero for rigid surfaces); $k=$ $/c$, where c is the speed of sound; $z_0=$ $_0c$, where $_0$ is the mean density; G and G' are the free-space Green's function and its normal derivative, respectively. The coefficient $C(P)$ is a constant, whose value is given as follows:

a. for interior problems :

$$C(P) = 4\pi \qquad \text{for } P \text{ inside of } S$$

$$C(P) = \int_S \frac{\partial}{\partial v}\left[\frac{1}{R(P,Q)}\right] dS(Q) \qquad \text{for } P \text{ on } S \qquad (2)$$

$$C(P) = 0 \qquad \text{for } P \text{ outside of } S$$

b. for exterior problems :

$$C(P) = 4\pi \qquad \text{for } P \text{ outside of } S$$

$$C(P) = 4\pi + \int_S \frac{\partial}{\partial v}\left[\frac{1}{R(P,Q)}\right] dS(Q) \qquad \text{for } P \text{ on } S \qquad (3)$$

$$C(P) = 0 \qquad \text{for } P \text{ inside of } S$$

In Eqs. (2) and (3), ν is the outward unit normal on S. The interior problems are those concerned with the acoustics of an enclosure (cavity) bounded by S, while the exterior problems are those concerned with the acoustics in infinite domain outside of (bounded by) S.

For muffler problems we use multi-domain BEM wherein the chambers and tubes are divided into "domains" to facilitate the modeling using boundary elements. Cheng, Seybert and Wu (1991) described extensively the applications of the BEM and multi-domain BEM. The BEM modeling does not require an end correction to elements such as extended inlets and outlets as with plane-wave and lump-parameter methods. With the BEM, the surface of each domain is modeled using nodes and two-dimensional elements. The resulting set of algebraic equations is solved to determine the unknown sound pressure and/or particle velocities. Most BEM models of mufflers can be solved on a PC.

Often, dissipative elements such as porous materials and perforated tubes are used in muffler design. Using multi-domain BEM, modeling involving these elements can be conveniently conducted. The acoustic impedance of the porous material replaces the rigid wall boundary condition on those surfaces where the porous material is placed. If the thickness of the porous material is not small relative to a wavelength, the porous material will allow waves to propagate parallel to the surface. In this case, the material is called a bulk-reacting material, and the complex characteristic impedance of the material is used instead of the local acoustic impedance. The BEM can model the bulk-reacting material of any thickness even when the material fills the entire cavity of the muffler.

Many applications of muffler design involve perforated tubes as dissipative elements because they are immune to the adverse effects of flows such as heat, moisture, and the presence of particulate. Moreover, by combining with a backing cavity, perforates may be used to create a muffler having a broad-band attenuation. Perforated tubes are modeled using a transfer impedance that depends on the hole size d_h, porosity (percent of open area due to the holes) , wall thickness *t*, and most important, the Mach number *M* of the flow. Expression of the relationship involving those parameters is as follows (Munjal, 1987):

$$z_t = [7.337\times10^{-3}(1+72.23M) + j2.2245\times10^{-5}(1+51t)(1+204d_h)f]p_0 c/\sigma \qquad (4)$$

where *f* is the frequency.

3. Examples

The following example shows an analysis of a simple expansion chamber of 152.4 mm diameter, in which the result of the BEM calculation, using the boundary element program BEMAP, is presented along with plane-wave theory prediction and experimental data. Figure 1 depicts the transmission loss of the expansion chamber in which the BEM solution is compared with the experimental result. The transmission loss of a muffler model can be derived in terms of the four-pole parameters (Igarashi and Toyama,1958) by assuming that the outlet is non-reflecting and the inlet and outlet cross-sectional areas are the same. It can be seen that the BEM and the experimental result agree very well.

The prediction from the plane-wave theory is shown in Fig. 2 wherein the experimental result is also presented for comparison. It can be seen that the plane-wave theory fails at frequencies above 2700 Hz due to the onset of higher-order acoustic modes in the chamber. On the other hand, the BEM works well in the whole frequency range as shown in Fig. 1. The behavior of the wave propagation may be observed in Fig. 3, where the sound contour is depicted at frequency of 2900 Hz. This is one of the frequencies at which the higher order acoustic modes occur. The sound contour shows that the waves propagate through, unimpeded, to the outlet. Thus, the higher order modes "short circuit" the chamber, and the sound waves pass through. The plane-wave

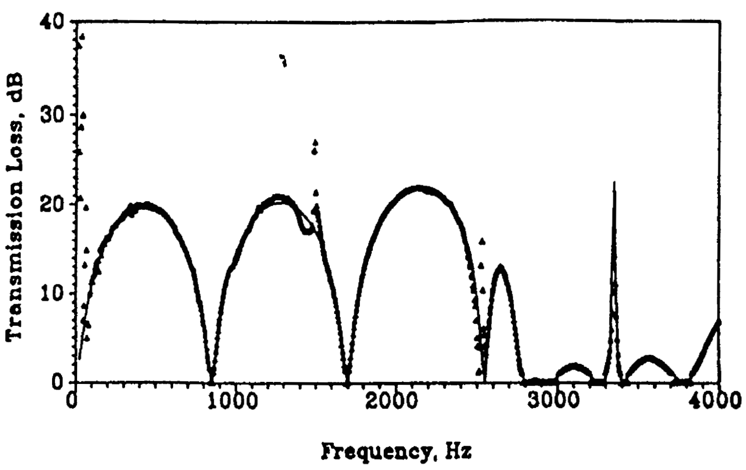

Fig. 1. Transmission loss of a simple expansion chamber of length 203 mm, diameter 152.4 mm, and expansion ratio 19: solid line, BEMAP solution; symbols, experiment.

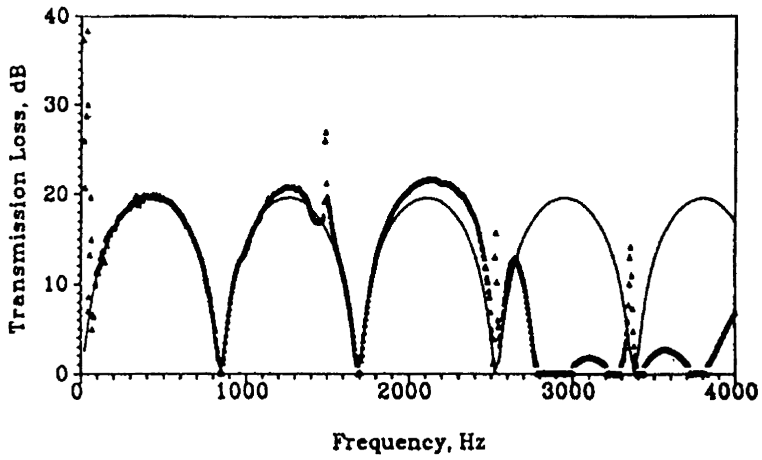

Fig. 2. Transmission loss of a simple expansion chamber described in Fig. 1: solid line, plane-wave theory; symbols, experiment.

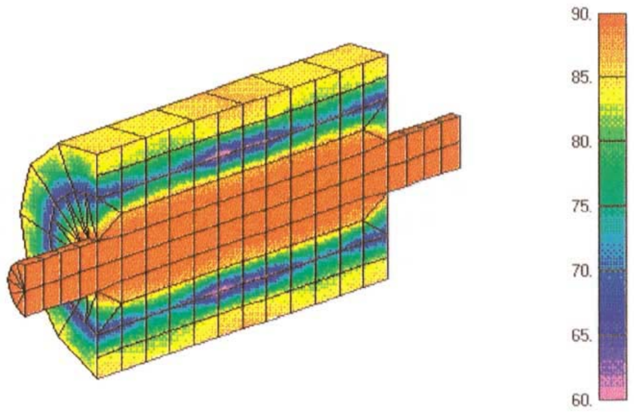

Fig. 3. Predicted sound pressure level (dB) contours inside the muffler at 2900 Hz using BEM.

theory cannot predict this phenomenon.

Another example of the occurence of the higher order modes is shown in Fig. 4 at 3800 Hz. One can observe how this mode causes the sound to pass through the muffler without being attenuated and therefore the muffler fails to perform at this frequency. This phenomenon is predicted well by the BEM as shown in Fig. 1, but not by the plane-wave theory.

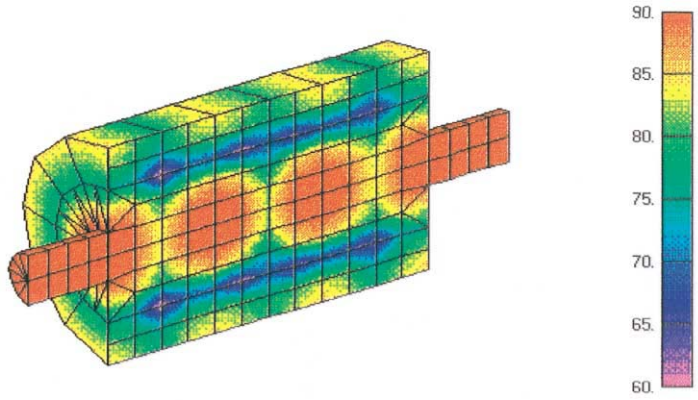

Fig. 4. Predicted sound pressure level (dB) contours inside the muffler at 3800 Hz using BEM.

Figure 5 shows the sound contour at 421 Hz. One may observe that at this frequency, plane wave propagates through the chamber, and therefore the plane-wave theory is applicable. The value of the transmission loss at 421 Hz, as seen from Fig. 2, is around 20 dB, wherein the result of the plane-wave theory agrees well with those of the BEM and experiment.

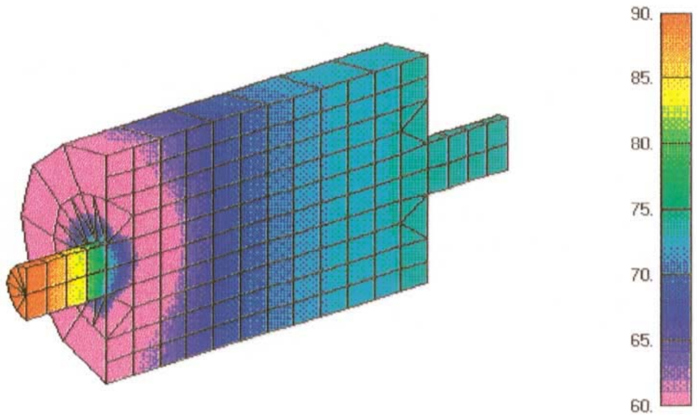

Fig. 5. Predicted sound pressure level (dB) contours inside the muffler at 421 Hz using BEM.

Figure 6 depicts the behavior of the waves propagating in the muffler at 850 Hz. The plane-wave theory is applicable at this frequency; as can be seen from this figure that the propagating wave is indeed a plane-wave. The plane-wave prediction agrees well with the BEM results. The muffler does not perform well at this frequency due to the resonance frequency of the chamber, in which an antinode occurs at the outlet of the muffler, because of the enforcement of the reflected waves in the chamber. It can be observed from Fig. 6 that no attenuation is obtained by the muffler.

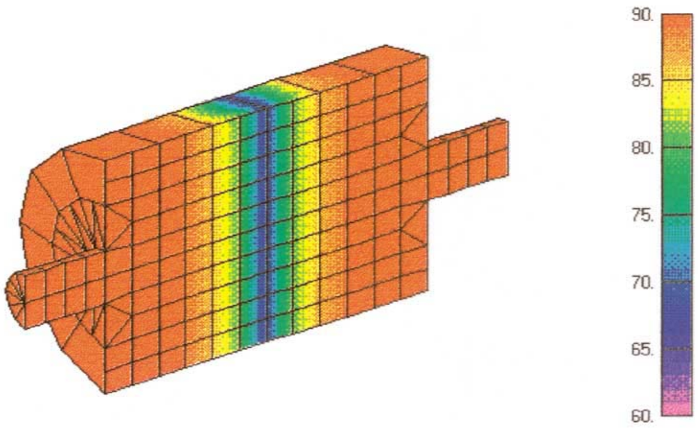

Fig. 6. Predicted sound pressure level(dB)contours inside the muffler at 850 Hz using BEM.

A concentric-tube resonator is shown in Fig. 7 consisting of a perforated tube through the center of an expansion chamber of the same dimensions as that analyzed in Figs. 1 and 2.

Due to the complex interaction between sound waves in the tube, in the perforate, and in the expansion chamber, plane-wave theory is not able to predict the performance of the concentric-tube resonator accurately. However, with the multi-domain BEM, such problems can be conveniently solved. By considering the tube as one domain and the expansion chamber surrounding the tube as a second domain, the transmission loss of this resonator can be determined. The transfer impedance between domains was represented by a formula similar to Eq. (4) for no flow (Munjal, 1987). The transmission loss of the concentric-tube resonator with a hole porosity = 13.4 percent is shown in Fig. 8.

It can be seen that good agreement was obtained between the BEM and the experimental results, except near the peak at 2300 Hz where a low signal-to-noise ratio prevented accurate measurement from being made.

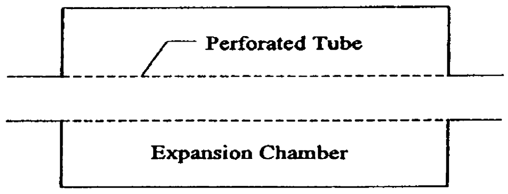

Fig. 7. A concentric-tube resonator.

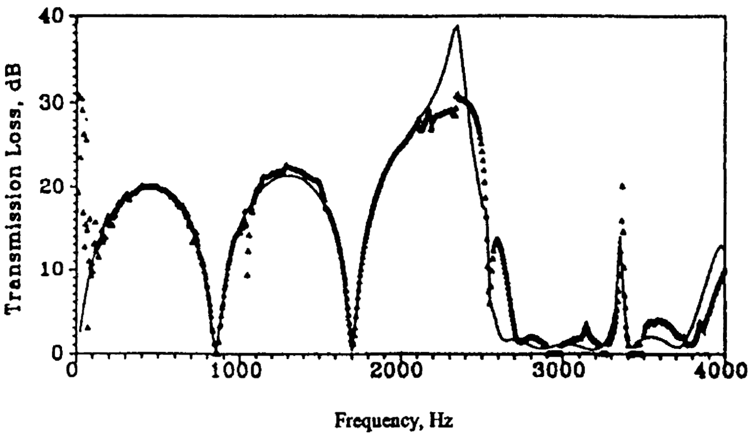

Fig. 8. Transmission loss of the concentric-tube resonator in Fig. 5 with σ = 13.4 percent: solid line, BEMAP; symbols, experiment.

4. Conclusions

A Boundary Element Method (BEM) for muffler performance calculation has been presented. It is shown that, by using the results obtained by the BEM, the behavior of the sound propagation inside the muffler can be visualized to explain how the muffler performs. By using the BEM the transmission loss of the muffler can be predicted for all range of frequencies, even at high frequencies where higher order acoustic modes occur, at which the plane-wave theory fails.

A multi-domain BEM is an effective technique for the calculation of muffler performance, particularly for complex muffler configurations such as those consisting of perforated tubes. By using the BEM, muffler designers may be able to develop mufflers with better high-frequency performance.

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Author Profile

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Andrew F. Seybert : He is professor of Mechanical Engineering at the University of Kentucky where he has been a faculty member since 1977. He received the B.S. degree from the University of Cincinnati and the M.S. and Ph.D. degrees from Purdue University. His research specialization is acoustics and vibrations. He is a Fellow of the Acoustical Society of America. At the University of Kentucky he has served as Director of Graduate Studies for the Department of Mechanical Engineering and Chair of the Mechanical Engineering Department from 1988-94. During 1994-95 He spent a sabbatical year at the Technical University of Berlin. In 1996 He helped organize the Vibro-Acoustics Consortium, a group of companies interested in the development and use of noise prediction technology. He has served as Vice-President of the ASME Noise Control and Acoustics Division and was Program Chairman for the 1994 Winter Annual Meeting. He is a registered Professional Engineer in the states of Kentucky and Ohio.